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Dynamic and Acoustic Characteristics of Bell Type Structure Using Finite Element Method

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Dynamic and acoustic characteristics of the bell type structure were analyzed numerically. The finite element method with 3-D general shell element was used to identify the natural frequencies and mode shapes of the structure. Mode shapes and stress distributions of a transient dynamic analysis were effectively displayed by using computer graphic technique. The results of this numerical study were in good agreement with those obtained from the experimental test of fast Fourier transform analyer. Vibrational modes which effect the acoustic characteristics of the typical bell-type structure were found to be the first flexural mode (4-0 mode) and the second flexural mode (6-0 mode). Asymmetric effects by Dangjwas and acoustic holes gave rise to beat frequencies, and the Dangjwa was found to be most effective. When the impact load was applied to the bell, the stress concentration occured at the rim part of the bell. It was found that the bell type structure should be designed thickly at the rim part in order to prevent failure from impact loads.

Key Words: Dynamic Behavior, Acoustic Behavior, Bell Type Structure, Finite Element Method, Transient Dynamic Analysis

1. Introduction

The bell type structure has been used as a timing signal regardless of the East or the West. Whereas the West bell, like a church-bell, usually sounds several times in a short moment. The East bell including Korean has characteristics transmits the sound far away after impacting and the artistic aspect of the sound itself was considered to be very important.

The West bells mostly are of small size, outside curved shape and are made of steel and bronze.On the other hand, the East bells relatively are of big size, inside round shape, asymmetry and are made of bronze material.

The dynamic characteristics of the bell type structure was studied by Rayleigh (1887) who assumed that the modes of low natural frequencies mostly occurred at the field of bending. Around the late 1920's to 1930's, the West bell was studied by Jones (1928) and Curtiss (1933, 1935) as a shell structure. The study of the bell structure had been further investigated by Timoshenko (1959), Warbuton (1949), Kalins (1964) and Leissa(1978). On the other hand, the East bell with importance of the sound was studied by Aoki and Yamashita (1932, 1948) in Japan. In these studies, they mainly investigated the natural frequency of the bell structure with the experimental tests. In 1970, Komatsuzawoa (1970), through the extensive experiments, offered several data which will be useful in the design of the bell. Rencently, finite element method (FEM) using computers was applied to the dynamic characteristic study of the bell structure. The dynamic characteristics of the Korean Yi-dynastic bells

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were studied numerically in Korea (Chung, 1985), but Ansari (1983) had not considered asymmetric effects. Also Yum (Chung, et al., 1985, 1986; Ansari, 1983; Yamashita, et al., 1932; Yum and Kim, 1981; Yum, Kwak and Chang, 1982; Yum, Lee and Chung, 1982) et. al. used the finite element method. But the accuracy of the solution was not good because they used plate elements. Chung and Kong (1986) appeared to be the first who attempted the use of 3-dimensional FEM in the study of dynamic characteristic of the bell type structure. However, the dynamic stress needs further in-depth study.

Accordingly, in this paper the analysis of eigenvalues and modes was performed, in which the shape effect of external shape, inside round shape, thickness as well as the asymmetric effect of Dangjwas and acoustic holes of the bell type structure were considered. Moreover, the stress distribution of the bell structure and the allowable strength in case of the impact load, which was based on eigenvalues analysis were analysed. The experimental modal analysis evaluated the accuracy of the program used in this study.

2. The Process of Analysis

2.1 The theory of linear dynamic analysis (EMRC, 1990)

In this study, FEM is used in order to analyze the dynamic behavior of the Korean bell with an asymmetric and complicated curved shape.

2.1.1 Eigenvalue analysis

The element type of finite method applied in this study is a 3-dimensional general shell element with both membrane stiffness and bending stiffness including transverse shear flexibility. The element has 6 degrees of freedom (UX, UY, UZ, ROTX, ROTY, ROTZ) at each node, but it does not have the rotational stiffness around the normal direction at the shell surface.

The coordinate system is shown in Fig. 1. The displacement vector{u} of each node at the element consists of inplane displacements u and v by membrane stresses,the deflection w by bending

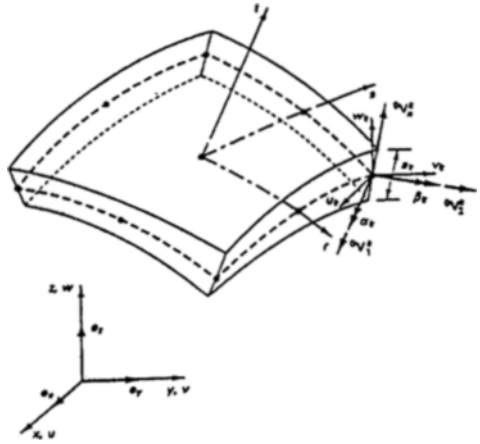

Fig. 1 Coordinate systems for 3-D general shell element

stresses and rotations α and β.

The generalized governing equation of the dynamic system may be written as

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]u= f(t) \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices, respectively and {u} and {f(t)} are the displacement vector and the applied generalized force vector. For the eigenvalue problem, damping matrices and force vector, [C] and {f(t)}, are eliminated.

Assuming that the solution of the equation is $u = \bar{\varphi} e^{iwt+\psi}$,

$$(K-w^2M)\bar{\varphi}=0 \tag{2}$$

Substituting $\lambda=w^2$ into Eq. (2) yields

$$(K-\lambda M)\bar{\varphi}=0 \tag{3}$$

To have non-trivial solution,

$$\det(K-\lambda_i M)=0 \tag{4}$$

where $\lambda_i=w^2$ are eigenvalues and $\{\bar{\psi}\}$ are corresponding eigenvectors.

Coefficients of free vibration were defined by eigenvalues λ_i and mode shapes which are represented by eigenvector $\bar{\psi}$.

The method used in the analysis of eigenvalue problem is the accelerated subspace method (ASM)(EMRC, 1990). The ASM has the faster convergency than other methods and can get the solution without any difficulty even in the analysis of the approximate eigenvalues and the repeated eigenvalues.

2.1.2 Transient dynamic analysis

The transient dynamic analysis may be used to determine the response of structures subjected to arbitrary time-varying loads.

Uncoupled modal equations become

$$\{\ddot{q}_r\}+\begin{bmatrix}2\xi_1 w_1 & 0 & \cdots & 0\\ & 2\xi_2 w_2 & \cdots & 0\\ & & & \vdots\\ diag & & \ddots & 2\xi_r w_r\end{bmatrix}\{\dot{q}_r\}$$

$$+\begin{bmatrix}w_1^2 & 0 & \cdots & 0\\ & w_2^2 & \cdots & 0\\ & & \ddots & \vdots\\ diag & & & w_r^2\end{bmatrix}\{q_r\}=\{f_r(t)\}$$

$$=\Phi^T P(t) \quad (5)$$

where

$diag(2\xi_r w_r)=\Phi^T C\Phi$, $diag(w_r^2)=\Phi^T T\Phi$

$\Phi=[\varphi_1, \varphi_2, \ldots, \varphi_n]$

ξ_r is a damping ratio for the rth mode, Φ is an n×m model matrix, and $\{f_r(t)\}$ is the forcing function for mode r. For the underdamped case ($\xi_r<1$), the solution of the typical modal Eq. (5) is

$$q_r(t)=e^{-\xi_r \omega_r t}[a_r Sin\bar{w}_r t+\beta_r Cos\bar{w}_r t] + \int_0^t f_r(\tau)h_r(t-\tau)d\tau \quad (6)$$

where $h_r(t-\tau)$ is the unit-impulse response function. α_r and β_r are constants evaluated from initial conditions and $\bar{w}_r=w_r\sqrt{1-\xi_r^2}$.

From the above equation, the solutions for the elastic modes are obtained as

$$\begin{bmatrix}q_r\\ \dot{q}_r\end{bmatrix}_{n+1}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}q_r\\ \dot{q}_r\end{bmatrix}_n + \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}\begin{bmatrix}f_{r,n}\\ f_{r,n+1}\end{bmatrix}, (n=0, 1, 2, -) \quad (7)$$

$$\ddot{q}_n=f_{r,n}-2\xi_r w_r \dot{q}_n - w^2{}_r q_n, \ (n=0, 1, -) \quad (8)$$

From Eqs. (7) and (8), the displacement, the velocity and the acceleration can be obtained

2.2 The finite element modeling of bell structure

The bell used in this study has subscale of 40 percents of the real size of Korean Yi-dynasty bell. The analysis model is shown in Fig. 2.

To obtain the effect of wall thickness, three kinds of model cases were used. The 'Case 3' model(t_{ave}=14mm) is slightly thicker than the 'Case 2' model(t_{ave}=10mm) and the 'Case 2' model is thicker than the 'Case 1' model(t_{ave}=6mm).

To investigate the effect of the rim of the bell, the thickness of the 'Case 2' model was changed only at the rim part of the circumferential direction. The 'Case 2-l' model, the 'Case 2-m' model, and the 'Case 2-n' model have incrementally increased thickness at the rim part of the 'Case 2' model, by 5mm, respectively.

To analyze asymmetric effects such as Dangjwas and acoustic holes, Dangjwas, which had the 10mm thickness, have attached to the 'Case 2' model, 'Case 2-1' model and 'Case 2-2' model have one and two symmetrically located Dangjwas, respectively. 'Case 2-A' model and 'Case 2-B' model had one Dwangjwas and the Dwangjwas's thickness was 5mm and 15mm, respectively.

'Case 2-Ⅰ', 'Case 2-Ⅱ' and 'Case 2-Ⅲ' was built by eliminating four elements, two elements and one element, respectively, from the 'Case 2' in order to produce holes.

The mesh was generated so that it is close to the orignal bell shape and to minimize the computing

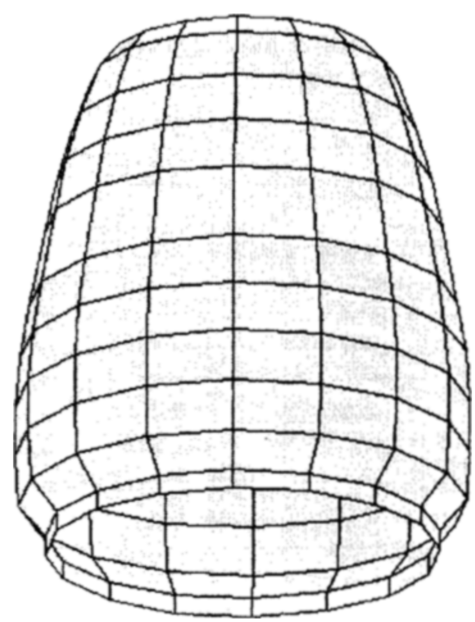

Fig. 2 Bell structure used in this study

Table 1 Dimensions and material properties of the test bell

Dimensions			
Height	Top-Dia.	Max-Dia.	Rim-Dia.
416mm	231mm	342mm	297mm

Material Properties				
Material type (Bronze)	Young's Modulus (GPa)	Poisson's Ratio	Weight Density (kg/m³)	Tensile Strength (MPa)
18% Sn	88.5Gpa	0.34	8600	429.1

time. After the mesh generation, total number of elements and nodes were 208 and 218 with 16 elements in the circumferential direction and 14 elements in the meridional direction. On the other hand, for 'Case 2- I ', 'Case 2-II' and 'Case 2-III' some elements were removed, as mentioned before. Total number of elements and nodes at these cases were 204 elements, 206 elements, 207 elements and 217 nodes, 218 nodes, 218 nodes respectively.

As a boundary condition, the hanging point was fixed in 6 degrees of freedom, but all other points were free.

The material of the bell used in this study was bronze. The dimensions and the mechanical properties were shown in Table 1.

2.3 The evaluation of the methods used in this study

The method used in this study was verified of comparing the theoretical results from reference papers (Ansari, 1983) and the experimental test results.

2.3.1 The evaluation of eigen-value problem analysis

To evaluate the accuracy of the solution obtained from the study, the known solutions of eigenvalue problem were compared to the results of this study.

The structural shape and material properties used in this study are shown in Fig. 3.

Table 2 shows dimensions and material properties of clamped-free conical shell.

Table 3 shows the comparison of natural frequencies obtained by using diffrent methods. It was found that the natural frequencies agreed well.

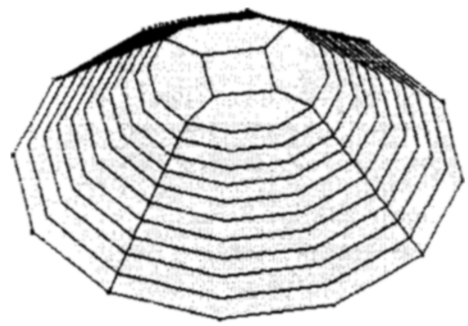

Fig. 3 Clamped-free conical shell

Table 2 Dimensions and material properties of clamped-free conical shell

Dimensions			
Thickness	Top-Dia.	Bottom-Dia.	Height
0.6mm	102mm	254mm	44mm

Material Properties		
Young's Modulus(GPa)	Poisson's Ratio	Density (Kg/m³)
1.04	0.25	320

Table 3 Comparision of natural frequencies

Mode	Proposed Method	E. P. Popovetal method	J. Ansari Method
1st	1072 Hz	1072 Hz	1071 Hz
2nd	1313 Hz	1315 Hz	1316 Hz
3th	1655 Hz	1611 Hz	1612 Hz

2.3.2 The evaluation using the experimental test

The purpose of the experimental work was to verify the validity of the computational results described in the previous section.

Experimental setup used in this experiment were F. F. T analyzer (RION SA-74), an impulse hammer (RION PH-51), an accelerometer (RION PV-95), and an amplifier (RION 2ch ANP VP-38).

The F. F. T analyzer obtains the input [F(jw)] of impulse through the load sensor attached to an impulse hammer. The output [X(Jw)] is obtained by the acceleration sensor attached to the structure. Using harmonic input [F(Jw)] and output [X(Jw)], the transfer function [H(Jw)] is

Fig. 4 Bell and instruments used for experimental test

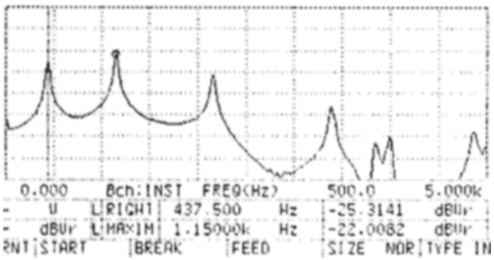

Fig. 5 Natural frequency by F. F. T. test

obtained as follows:

$$H(Jw)=\frac{X(Jw)}{F(Jw)} \tag{9}$$

The bell type structure used in the experimental test, which has the height of 163mm, the top diameter of 76mm, and the max diameter of 120mm. It is shown in Fig. 4 and the material properties of this bell are given in Table 1.

The natural frequencies acquisited by F. F. T analyzer are shown in Fig. 5. Table 4 shows that the experimental results and the theoritical results obtained by the method proposed in this study.

The first, second and third pick point at the Fig. 5 meant the frequency of the first mode, second mode, and third mode of the bell shown in Fig. 4, respectively.

Table 4 Comparison of natural frequencies obtained by F. F. T experimental test and the proposed method

Flexural Mode No.	F. F. T. Test	This Study	Mode Shape
1st Mode	437.5Hz	430.9Hz	4-0
2nd Mode	1.15KHz	1.23KHz	6-0
3rd Mode	2.21KHz	2.45KHz	8-0

3. The Structure Analysis of the Bell

The structure analysis of the bell used in this study consisted of two types. One was the natural frequency and vibration shape of each mode which could be found by eigenvalue analysis. The other was the analysis of stress distribution and determination of the maximum impact load that does not destroy the bell.

3.1 Eigenvalue anlysis

In this study, the effects of wall thickness, asymmetric Dangjwas, and acoustic holes to find the acoustic and structure character istics were considered.

3.1.1 Effects of wall thickness

The natural frequencies and mode shapes are tabulated in Table 5. It was found that the bell with the thicker wall had higher natural frequencies. But vibration mode shapes were similar to each other. Mode shapes of 'Case 3' are shown in Fig. 6. Mode shapes were defined as 4-0, 6-0, 8-0 and so on. Here, for instance, '4' means the circumferential mode shape, and '0' means the longitudinal mode shape.

3.1.2 Rim thickness effect

The Korean bell typically has a thick rim. Table 6 shows the variation of the natural frequencies as the thickness of the rim varies. From the above results, it was found that the thicker rim had the higher frequency and vice versa. Therefore, a designer can easily adjust the frequencies by changing the thickness of the rim, even after the bell is casted.

3.1.3 Asymmetric effect of the Dangjwas and acoustic hole

Table 7 shows the natural frequencies and beat frequencies produced by the asymmetric effect of the Dangjwas, which is the impacting point of the bell.

From the results shown in Table 7, it was found that Dangjwas produced the beat phenomena, with two Dangjwas (the interval of 180°) having bigger beats than the others. The natural

Table 5 Comparison of natural frequencies by the wall thickness change of bell

Flexural Mode No.	Case 1 (6mm)	Case 2 (10mm)	Case 3 (14mm)	Mode Shape	
				Axi p	Cir q
1st Mode	180.2Hz	247.4Hz	312.9Hz	0	4
2nd Mode	580.5Hz	769.0Hz	950.7Hz	0	6
3rd Mode	1259.9Hz	1477.5Hz	1858.2Hz	0	8

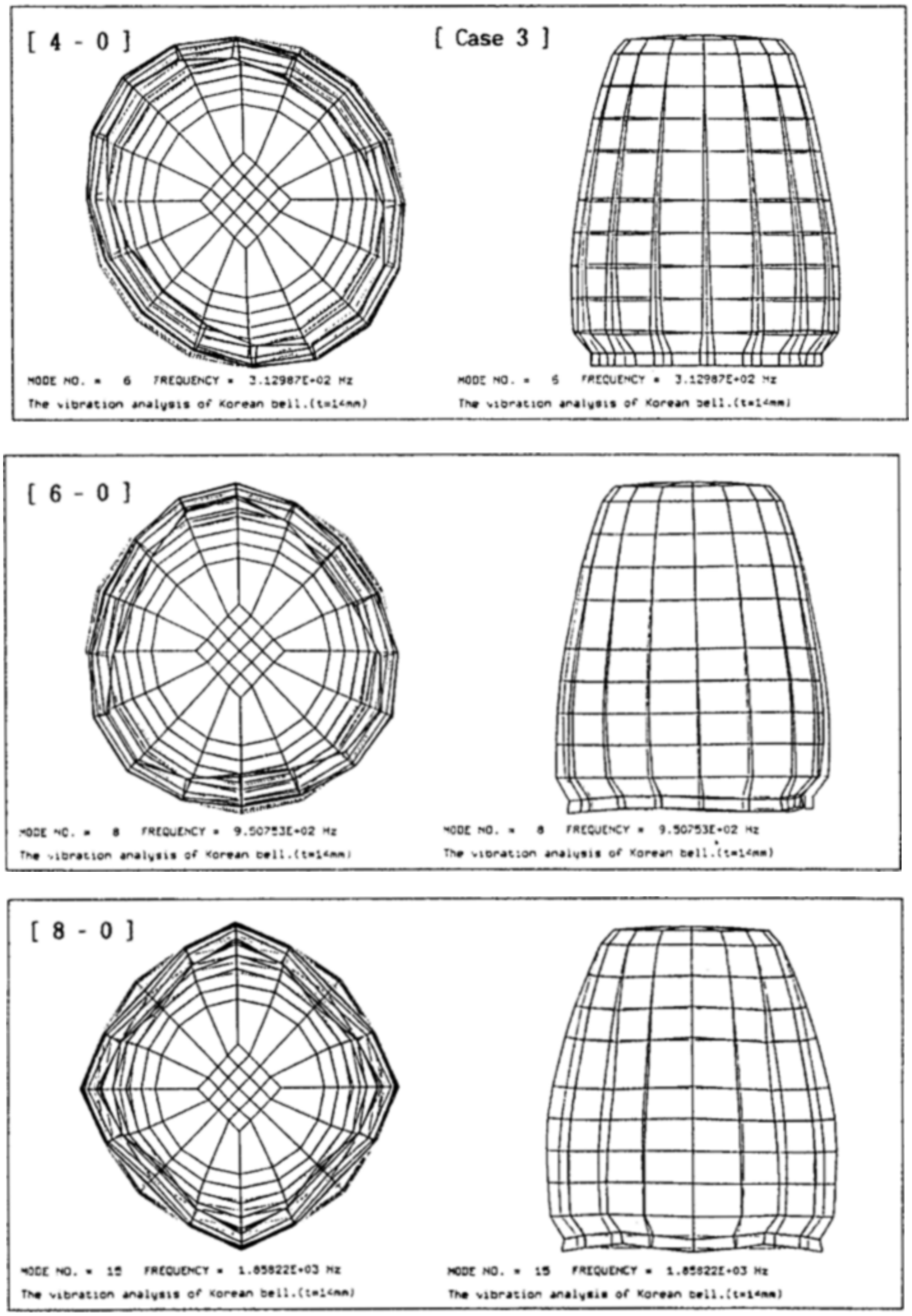

Fig. 6 3-D Mode shapes of case 3 without asymmetric effect

Table 6 Comparison of natural frequencies by the thickness change of rim

Flexural Mode No.	Case 2	Case 2-l (15mm)	Case 2-m (20mm)	Case 2-n (25mm)	Mode Shape	
					Axi. p	Cir. q
1st Mode	247.4Hz	273.3Hz	310.5Hz	355.9Hz	0	4
2nd Mode	769.0Hz	848.1Hz	939.8Hz	1014.6Hz	0	6
3rd Mode	1477.5Hz	1507.2Hz	1521.3Hz	1527.9Hz	0	8

Table 7 Variation of natural frequencies and beat frequencies by the effect of dwangjwas and thickness change

Flexural Mode No.		Case 2	Case 2-1 (1 Dangjwa)		Case 2-2 (2 Dangjwa)		Case 2-A (5mm)		Case 2-B (15mm)	
			N.F.	B.F.	N.F.	B.F.	N.F.	B.F.	N.F.	B.F.
1st Mode	Low	247.4Hz	247.486Hz	0.255	247.533Hz	0.508	247.450Hz	0.18	427.514Hz	0.19
	High		247.741Hz		248.041Hz		247.630Hz		247.704Hz	
2nd Mode	Low	769.0Hz	768.600Hz	1.685	768.145Hz	3.4	768.840Hz	0.999	768.328Hz	1.944
	High		770.285Hz		771.554Hz		769.839Hz		770.272Hz	

(N. F. Natural Frequency, B. F: Beat Frequency)

Table 8 Variation of natural frequencies and beat frequencies by the hole size of bell

Flexural Mode No.		Case 2-Ⅰ (Large)		Case 2-Ⅱ (Middle)		Case 2-Ⅲ (Small)	
		N.F.	B.F.	N.F.	B.F.	N.F.	B.F.
1st Mode	Low	247.42Hz	0.01	247.42Hz	0.02	247.42Hz	0.02
	High	247.43Hz		247.44Hz		247.42Hz	
2nd Mode	Low	769.04Hz	0.00	769.05Hz	0.00	769.05Hz	0.00
	High	769.04Hz		769.05Hz		769.05Hz	

(N.F:Natural Frequency, B.F:Beat Frequency)

frequencies and beat frequencies produced by the acoustic hole are shown in Table 8. But the beat frequencies were not generated by adding acoustic holes.

3.2 Transient dynamic analysis

'Case 2' has 10mm wall, 'Case 2-m' has 10mm wall with 10mm rim, 'Case 2-1' has 10mm wall and one Dangjwa, and 'Case 2-III' has 10mm wall and one hole. The impulse load was applied to the Dangjwa of the Case 2-1. The impulse load for the Cases without the Dangjwa also acted on the same position.

The coordinate of the position which the impulse load acted on was (-60.0, 194.7, 160), (0.0, 167.5, 160.0), (-61.6, 148.7, 200.0) and (0.0, 161.0, 200.0).

Figure 7 shows the time interval of an impulse

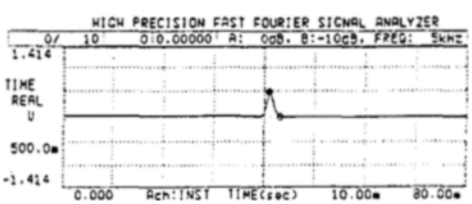

Fig. 7 The time interval of impulse load about the bell

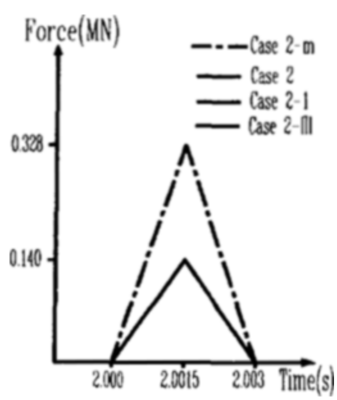

Fig. 8 Force-time history applied to this study

load which acted on the bell through the impulse hammer experiment. Force-time history of the impulse load from Fig. 7 was shown in Fig. 8.

The yield stress of the test bell was 450Mpa. The maximum stress by the impact load and stress distributions along time in 'Case 2-m' and 'Case 2-1' is shown in Fig. 9 and Fig. 10, respectively. When the time interval of the impact load in 'Case 2-m' was changed to five milli-second, the stress history of the impact load was shown in Fig. 11. Here, it was found that as the impact load time was closer to the natural cycle, the higher

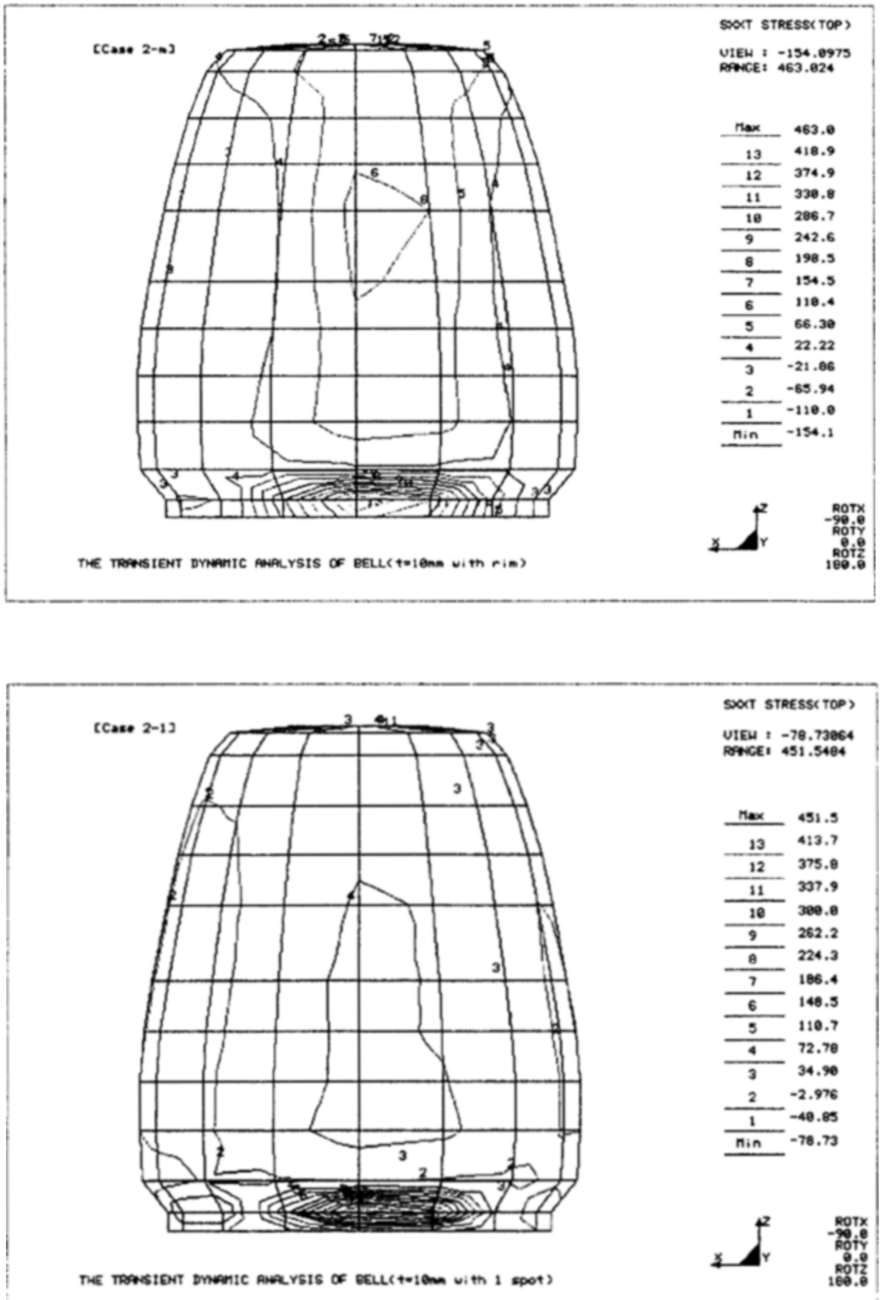

Fig. 9 Maximum stress by thei impact load for the Case 2-m and the Case 2-1

stress history was produced.

Impact failure loads and the maximum stresses in four models of bell are shown in Table 9. As a result, it was found that the stress concentration occurred at the rim of the bell. Therefore, the rim part should be thickened in order to prevent failure from impact loads catapulated by striking.

Table 9 Impluse failure load and maximum stress by transient dynamic analysis of bell

	Failure Load	Max-Stress
Case 2	1.40×10^6 N	463 Mpa
Case 2-m	3.28×10^6 N	463 Mpa
Case 2-I	1.40×10^6 N	451 Mpa
Case 2-Ⅲ	1.40×10^6 N	463 Mpa

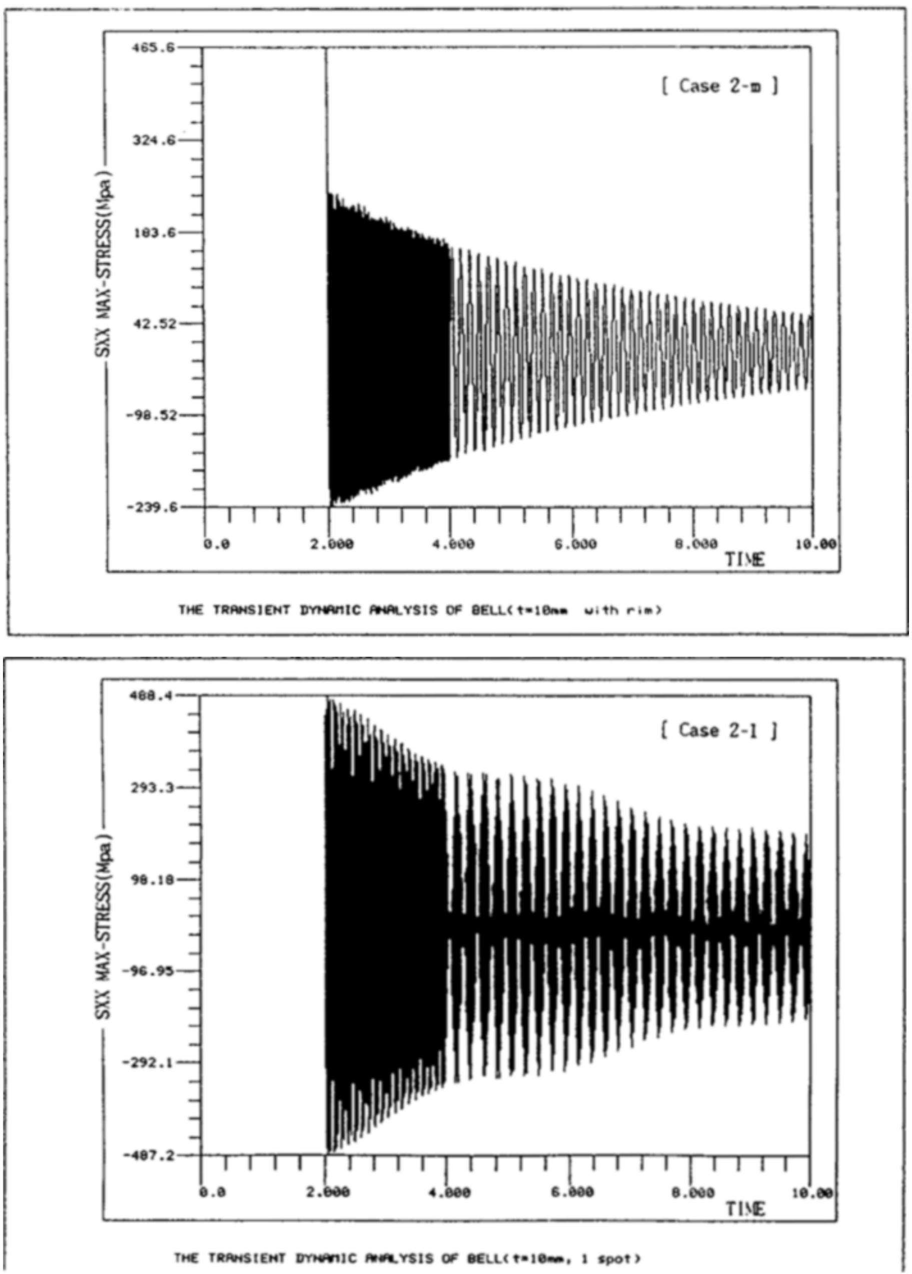

Fig. 10 Stress distributions along time about the Case 2-m and the Case 2-1

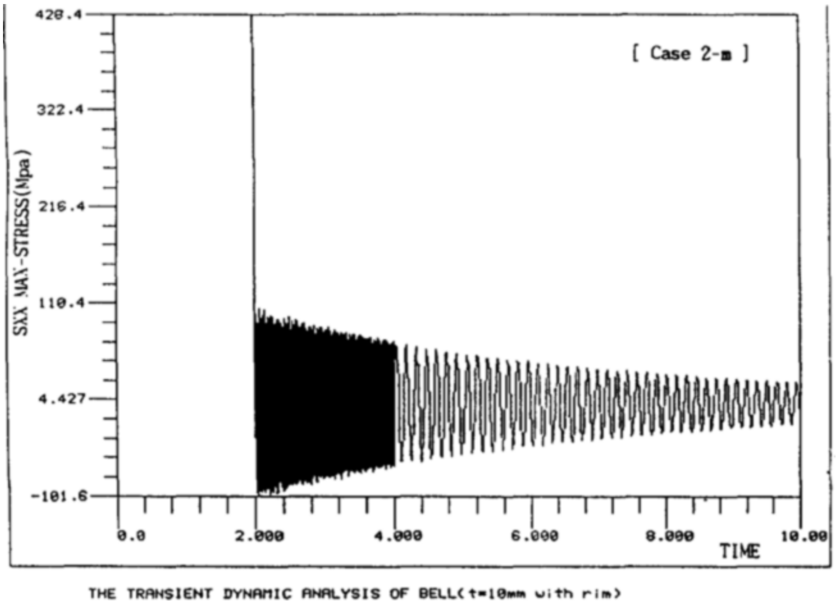

Fig. 11 Stress distribution along time about Case 2-m (5msec)

4. Conclusions

In this study, the dynamic behaviors of the Korean bell type structure in light of the asymmetric effect were studied and followings were deduced from various experiments;

(1) Natural vibration modes mostly depend on the first to third fundamental frequencies of flexural modes. Mode shapes were defined as 4-0mode, 6-0mode, and 8-0mode, which are the source of the lingering sound.

(2) After an impact, the bell with the thicker wall and rim not only had higher natural frequencies,but also had larger increment of frequencies.

Therefore, the bell should be designed thick in order to produce high pitched sounds. Natural frequencies can be adjusted easily by changing the thickness of the rim without big modifications.

(3) Asymmetric effects, such as Dangjwas and acoustic holes, caused beat frequencies, which were increased by increasing the number and thickness of Dangjwa. Whereas, the change of acoustic hole size had a little effect on the beat frequency.

(4) In transient dynamic analysis, the range of impact load that could not fail the bell and the displacement and stress distribution were calculated. The bell was structurally safe if we design it be as thick as possible at the rim part. Therefore, the bell type structure should be designed thick at the rim part in order to prevent failure from the impact loads launched by striking.

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